E 7616 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2019

First Semester

Complementary Course—Mathematics—DIFFERENTIAL CALCULUS AND TRIGONOMETRY

(Complementary Course to Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control and Computer Maintenance and Electronics)

[Prior to 2013 Admissions]

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*

Each bunch of **four** *questions carries a weight of* 1.

I. 1 $\lim_{x\to 12}(10-3x)=\ldots$

2 $\lim_{x\to\infty}\frac{2}{x}-3=\ldots$

3 The curve $y=x^2-4x+4$ has a horizontal tangent at $x=$ _____.

4 If $y=\frac{3}{x}+5\sin x$, then $\frac{dy}{dx}=$ _____.

II. 5 The slope of the circle $x^2+y^2=25$ at (3, 4) is _____.

6 The absolute maximum value of $f(x)=x^2-1, -1\le x\le 2$ is at $x=$ _____.

7 If $f'(x)=0$ for all x and $f(-1)=3$, then for all x, $f(x)=$ _____.

8 If f is continuous on $[a, b]$ and if $f'(x)>0$ for every x in (a, b), then f is _____ on $[a, b]$.

III. 9 If $f'(x)=x^{-1/3}(x+2)$, find the critical points of f.

10 Show by an example (use graph of functions) that Rolle's theorem fails if $f(x)$ is continuous on $[a, b]$ but not differentiabe at an interior point.

11 Find $\frac{\partial f}{\partial x}$ if $f(x, y)=x^y$.

12 Find $\frac{\partial^2 f}{\partial y\,\partial x}$ if $f(x,y)=ye^x$.

Turn over

IV. 13 Write the chain rule formula for $\frac{dW}{dt}$ where $W = f(x, y, z)$ is differentiable and x, y and z are differentiable functions of t.

14 If $f(x, y)$ is differentiable and $f(x, y) = 0$ defines y as a differentiable function of x, then write a formula for $\frac{dy}{dx}$ at any point where $f_y \neq 0$.

15 Write the expression of $\cos x$ in terms of exponential functions.

16 What is the imaginary part of $\cos(ix)$, where x is real ?

$(4 \times 1 = 4)$

Part B (Short Answer Questions)

Answer any **five** *questions.*
Each question carries a weight of 1.

17 Let $f(x) = \begin{cases} x, & x < 1 \\ x+1, & x > 1 \end{cases}$. Show that $\lim_{x \to 1} f(x) \neq 1$.

18 The position of a particle at time t, moving along the s-axis is $s = t^3 - 6t^2 + 9t$ (in metres). Find the acceleration of the particle at $t = 3$ seconds.

19 State the Mean Value theorem.

20 As x moves from left to right through the point $c = 2$, is the function $f(x) = x^3 - 3x + 2$, increasing or decreasing ? Justify your answer.

21 Find the partial derivative f_{yxyz} if $f(x, y, z) = 1 - 3xy^2z + xy$.

22 Express $\frac{\partial w}{\partial r}$ in terms of r and s if $w = x + y + z^2, x = \frac{r}{s}, y = r^2 + s, z = r$.

23 Show that $\cos 4\theta = \cos^4\theta - 6\cos^2\theta \sin^2\theta + \sin^4\theta$.

24 Show that $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question carries a weight of 2.

25 Let $f(x) = \begin{cases} \sqrt{1-x^2}, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \\ 2, & x = 2 \end{cases}$

(a) What are the domain and range of f ?

(b) At what points c, if any, does $\lim_{x \to c} f(x)$ exist ?

(c) At what points does only the left hand limit exist ?

(d) At what points does only the right hand limit exist ?

26 Find $\frac{d^2y}{dx^2}$ as a function of t, if $x = t + \frac{1}{t}, y = t - \frac{1}{t}$.

27 Find the absolute maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-2, 3]$.

28 Find the critical points of f given that the derivative $f'(x) = (x-1)^2(x+2)$. Identify the intervals on which f is increasing and decreasing. At what points, if any, does f assume local maximum and minimum values ?

29 If $f(x,y,z) = (x^2 + y^2 + z^2)^{-1/2}$, show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$.

30 Expand $\sin^7 \theta$ in a series of sines of multiples of θ.

$(4 \times 2 = 8)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries a weight of 4.

31 (a) State Rolle's theorem. Apply the theorem to show that the equation $x^3 + 2x + 4 = 0$ has exactly one real root.

(b) Find the point c of mean value theorem for the function $f(x) = x^2 + 2x - 1$ in $0 \le x \le 1$.

32 (a) Find f_x, f_y and f_z if (i) $f(x,y,z) = \sin^{-1}(xyz)$; (ii) $f(x,y,z) = yz \log xy$.

(b) Evaluate $\frac{\partial z}{\partial u}$ at $(u,v) = \left(2, \frac{\pi}{4}\right)$ where $z = 4e^x \log y, x = \log(u \cos v), y = u \sin v$.

33 Sum to infinity the series :

$$1 + \frac{c^2 \cos 2\theta}{2!} + \frac{c^4 \cos 4\theta}{4!} + \ldots$$

$(2 \times 4 = 8)$